CLASS 12 – UP BOARD

MATHEMATICS – SET 7

ANSWERS key

खंड – A (MCQ with Concept)

1. $|5A^{-1}| = 5^3|A^{-1}| = 125 \times (1/5)$ = **25**

2. $\lim (1-\cos 3x)/x^2$
   $= ((3x)^2/2)/x^2$
   $= 9/2$

3. $\ln y = x \ln(x^2+1)$

4. $\int_0^\pi \cos^2 x \, dx = \pi/2$

5. $P(A \cup B) = 0.7 + 0.6 - 0.4 = 0.9$

6. $a \cdot b = (1)(2) + (2)(-1) + (3)(1) = 2 - 2 + 3 = 3$

7. $f'(x) = 4x^3 - 8x$
   $x=0$ पर $f'=0$
   $f''(0) = -8 < 0 \Rightarrow$ Local Maximum

8. $\int x/(x^2+9) \, dx$
   $= (1/2) \ln(x^2+9) + C$

9. $a \cdot b = |a||b|\cos 60° = 4 \times 3 \times 1/2 = 6$

10. $P(A \cap B) = (1/4)(2/5) = 1/10$

खंड – B (2 अंक – Proper Steps)

11. $(AB)^{-1} = B^{-1}A^{-1}$

$AB(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$
$= AIA^{-1} = I$
$\Rightarrow$ सिद्ध।

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12. $\lim (\tan 2x - 2x)/x^3$

$\tan 2x = 2x + (8x^3/3) + \ldots$

$\Rightarrow (8x^3/3)/x^3$

$= 8/3$

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13. y=ln(sinx)

dy/dx = cosx/sinx

= cotx

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14. ∫ e^x cosx dx

= e^x(sinx+cosx)/2 + C

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15. Independent Events

यदि P(A∩B)=P(A)P(B)

तो स्वतंत्र घटनाएँ।

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16. Scalar Triple Product

$$a \cdot (b \times c) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

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17. Increasing Function

यदि f'(x)>0 interval में

$\Rightarrow$ f(x) वर्धमान।

उदाहरण: f(x)=x^3

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18. ∫ (7x^6/(x^7+3))dx

Let t=x^7+3

dt=7x^6dx

$\Rightarrow$ ln(x^7+3)+C

खंड – C (4 अंक – Stepwise)

19. Inverse Matrix Method

हल करने पर:

x=1
y=2
z=3

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20. y=x²e^x

dy/dx=2xe^x+x²e^x
d²y/dx²=2e^x+4xe^x+x²e^x

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21. lim (e^x–e^a)/(x–a)

Derivative of e^x

= e^a

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22. ∫₀^π x cosx dx

By parts:

= –2

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23. यदि f'(x)>0

Slope positive ⇒ function increasing
पूर्ण प्रमाण derivative की परिभाषा से।

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24. दो रेखाओं के बीच कोण

$$cos\theta = \frac{a \cdot b}{|a||b|}$$

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25. dy/dx + y tanx = secx

Linear DE

IF=e^∫tanx dx = secx

⇒ y secx = ∫ sec²x dx
⇒ y secx = tanx + C
⇒ y = sinx + C cosx

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26. Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

पूर्ण प्रमाण।

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27. सिद्ध करें

$$\mid a \times b \mid^2 = \mid a \mid^2 \mid b \mid^2 - (a \cdot b)^2$$

Vector identity से सिद्ध।

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28. Maclaurin Series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots$$

खंड – D (6 अंक)

29. Cauchy Mean Value Theorem

यदि f,g सतत और अवकलनीय

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

f(x)=x², g(x)=x³ पर लागू।

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30. ∫ x²e^x dx

= e^x(x²−2x+2)+C

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31. Gauss Elimination

Row reduction से:

x=2
y=1
z=3

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32. Skew Lines Distance

$$D = \frac{|\,(a_2 - a_1) \cdot (b_1 \times b_2)\,|}{|\,b_1 \times b_2\,|}$$

पूरा determinant प्रमाण।

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33. dy/dx=(x+y)/(x−y)

Homogeneous DE

Let y=vx

हल करने पर:

x²−y²=Cx