CLASS 12 – UP BOARD

MATHEMATICS – SET 4

ANSWERS KEY

खंड – A (MCQ with Concept Reasoning)

1. $|3A^{-1}| = 3^3 |A^{-1}| = 27 \times (1/3) =$ **9**

2. $\lim \tan 5x/x = 5$

3. $dy/dx\ (x^x) = x^x(1+\ln x)$

4. $\int_0^1 1/(1+x^2)\ dx = \tan^{-1}x\ [0 \rightarrow 1] = \pi/4$

5. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
   $= 0.6 + 0.7 - 0.8 = 0.5$

6. $a \times b$ निकालकर परिमाण $= \sqrt{35}$

7. $f'(x) = 4x^3 - 12x^2 + 12x$
   $x=0$ पर $f'=0$
   $f''(0) = 12 > 0 \Rightarrow$ Local Minimum

8. $\int x/(x^2+4)\ dx$
   $= (1/2)\ \ln(x^2+4) + C$

9. $|a \times b| = |a||b|\sin 60°$
   $= 2 \times 3 \times (\sqrt{3}/2) = 3\sqrt{3}$

10. $P(A \cap B) = P(A|B)P(B)$
    $= (2/3)(3/5) = 2/5$

खंड – B (2 अंक)

11. adj(adj A) = |A|A

ज्ञात गुणधर्म:

$A(\text{adj } A) = |A|I$

दोनों पक्षों पर adj लें:

$\text{adj(adj } A) = |A|A$

सिद्ध।

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12. lim (sinx − x)/x³

sinx = x − x³/6 + ...

$\Rightarrow (-x^3/6)/x^3$
= −1/6

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13. y = tan⁻¹(x²)

dy/dx = (1/(1+x⁴))×2x
= 2x/(1+x⁴)

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14. ∫ e^(ax) dx

= (1/a)e^(ax) + C

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15. Bayes Theorem (Statement)

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{\sum P(A_j)P(B \mid A_j)}$$

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16. (a×b) ⊥ a और b

क्योंकि
(a×b)·a =0
(a×b)·b =0

⇒ लम्बवत

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17. Concavity

यदि f''(x)>0 ⇒ Concave Up
यदि f''(x)<0 ⇒ Concave Down

18. $\int \left(4x^3/(x^4+1) \right) dx$

Let $t=x^4+1$

$dt=4x^3 dx$

$\Rightarrow \ln(x^4+1)+C$

खंड – C (4 अंक)

19. Inverse Matrix Method

$AX=B$

A^{-1} निकालकर

हल:

$x=1$
$y=2$
$z=3$

(Determinant और adjoint विस्तार सहित)

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20. y=e^x sinx

$dy/dx = e^x \sin x + e^x \cos x$

$d^2y/dx^2 = e^x \sin x + 2e^x \cos x - e^x \sin x$
$= 2e^x \cos x$

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21. lim log(1+x)/x

$\log(1+x) \approx x$

$\Rightarrow$ limit $=1$

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22. $\int_0^\pi x \cos x \, dx$

By parts:

$= [x \sin x + \cos x]_0^\pi$
$= -2$

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23. यदि f''(x)>0

Slope बढ़ रही है ⇒ ग्राफ अवतल (concave up)
पूर्ण व्याख्या सहित।

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24. दो रेखाओं की न्यूनतम दूरी

$$D = \frac{|\,(a_2 - a_1) \cdot (b_1 \times b_2)\,|}{|\,b_1 \times b_2\,|}$$

पूर्ण वेक्टर प्रमाण।

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25. dy/dx + y/x = x²

Linear DE

IF=e^∫(1/x)dx = x

⇒ xy=∫ x³dx
⇒ y=x²/4 + C/x

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26. Conditional Probability Proof

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

पूर्ण प्रमेयात्मक प्रमाण।

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27. Scalar Triple Product

$$|\,a \cdot (b \times c)\,| = Volume\,of\,parallelepiped$$

पूर्ण ज्यामितीय प्रमाण।

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28. Taylor Series

$$f(x) = f(a) + (x - a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \dots$$

29. Taylor's Theorem

यदि f n बार अवकलनीय है:

$$f(x) = f(a) + (x - a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \dots$$

sinx का विस्तार:

$$sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

30. ∫ x³ e^x dx

By parts:

= e^x(x³−3x²+6x−6)+C

31. Gauss Elimination

Row operation से:

x=2
y=1
z=3

32. Skew Lines Distance

$$D = \frac{|\,(a_2 - a_1) \cdot (b_1 \times b_2)\,|}{|\,b_1 \times b_2\,|}$$

पूरा determinant प्रमाण सहित।

33. (x²+y²)dx−2xy dy=0

Rearrange:

$$\frac{dx}{2x} = \frac{dy}{y}$$

हल करने पर:

$x^2+y^2=Cx$

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34. Bayes Theorem (Advanced Example)

Medical Model:

Full stepwise substitution और गणना सहित।