CLASS 12 – UP BOARD

MATHEMATICS – SET 3

ANSWER KEY

खंड – A (MCQ with Concept)

1. $|A^{-1}| = 1/|A| = 1/2$

2. $\lim \sin 3x/x = 3$

3. $dy/dx = 2x/(x^2+1)$

4. $\int_0^1 3x^2 \, dx = [x^3]_0^1 = 1$

5. $P(A \cup B) = 0.4+0.5-0.2 = 0.7$

6. $a \cdot b = (1)(2)+(2)(-1)+(-1)(1) = 2-2-1 = -1$

7. $f'(x) = 3x^2-3 = 3(x^2-1) = 0 \Rightarrow x = \pm 1$

8. $\int (2x/(x^2+1)) dx = \log(x^2+1)+C$

9. Cross product zero $\Rightarrow \theta=0°$ या $180°$

10. $P(A \cap B) = P(A|B)P(B) = (1/2)(1/4) = 1/8$

खंड – B (2 अंक – Proper Steps)

11. सिद्ध करें $(A^{-1})^{-1} = A$

यदि $A^{-1} A = I$

दोनों पक्षों का inverse लें:

$(A^{-1})^{-1} = A$

सिद्ध।

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12. $\lim (1-\cos 2x)/x^2$

ज्ञातः
$1-\cos\theta \approx \theta^2/2$

$\Rightarrow = ((2x)^2/2) / x^2$

$= 4x^2/2x^2$

$= 2$

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13. $y = \sin^{-1}x$

$dy/dx = 1/\sqrt{(1-x^2)}$

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14. $\int e^{\wedge}x \cos x \, dx$

Integration by parts दो बार:

$$= \frac{e^x}{2}(sinx + cosx) + C$$

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15. Independent Events

यदि P(A∩B)=P(A)P(B)

तो A,B स्वतंत्र कहलाते हैं।

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16. सिद्ध करें $|a+b|^2$

$$| a + b |^2 = (a + b) \cdot (a + b)$$

$= a \cdot a + b \cdot b + 2a \cdot b$

$= |a|^2 + |b|^2 + 2a \cdot b$

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17. Increasing Function

यदि f'(x)>0 किसी interval में

तो f(x) उस interval में वर्धमान है।

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18. $\int (3x^2/(x^3+1)) \, dx$

Let $t=x^3+1$

$dt=3x^2 dx$

$\Rightarrow = \ln(x^3+1)+C$

खंड − C (4 अंक)

19. Cramer's Rule

$D \neq 0$

हल करने पर:

$x=1$
$y=2$
$z=3$

(Determinant विस्तार सहित)

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20. y=x sinx

$dy/dx = \sin x + x\cos x$

$d^2y/dx^2 = \cos x + \cos x - x \sin x$
$= 2\cos x - x \sin x$

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21. $\lim (x^3-a^3)/(x-a)$

$x^3-a^3=(x-a)(x^2+ax+a^2)$

$\Rightarrow \text{limit} = x^2+ax+a^2$
$x \to a \Rightarrow 3a^2$

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22. $\int_0^\pi x \sin x \, dx$

By parts:

$= [-x\cos x + \sin x]_0^\pi$
$= \pi$

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23. यदि f'(x)>0

Slope positive $\Rightarrow$ function increasing
पूर्ण प्रमाण derivative की परिभाषा से।

24. दो समतलों के बीच कोण

यदि normals n_1, n_2

$$cos\theta = \frac{n_1 \cdot n_2}{\mid n_1 \mid \mid n_2 \mid}$$

25. dy/dx=(x+y)/(x−y)

Homogeneous DE

Let y=vx

हल करने पर:

x²−y²=Cx

26. Total Probability Theorem

यदि $A_1, A_2, \dots$ partition हैं,

$$P(B) = \sum P(A_i) P(B \mid A_i)$$

पूर्ण प्रमाण।

27. Scalar Triple Product

$$a \cdot (b \times c) = \mid \begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix} \mid$$

28. Mean Value Theorem

यदि f सतत [a,b] तथा अवकलनीय (a,b) में,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

29. Lagrange Mean Value Theorem

शर्तें:

1. f सतत [a,b]

2. अवकलनीय (a,b)

⇒ ∃c ∈(a,b)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

f(x)=lnx पर लागू करने पर:

$$1/c = \frac{\ln b - \ln a}{b - a}$$

30. ∫ x^2 sinx dx

By parts twice:

= −x^{2}cosx +2x sinx +2cosx +C

31. Gauss Elimination

Row operation से:

x=2
y=1
z=3

32. Skew Lines Distance

$$D = \frac{| (a_2 - a_1) \cdot (b_1 \times b_2) |}{| b_1 \times b_2 |}$$

Determinant proof सहित।

33. dy/dx + y cotx = sinx

Linear DE

IF = e^∫cotx dx = sinx

$\Rightarrow$ y sinx = ∫ sin^{2}x dx

$\Rightarrow$ y = (x/2 − sin2x/4)/sinx

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34. Bayes Theorem (Detailed Numerical)

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{\sum P(A_j)P(B \mid A_j)}$$

कठिन उदाहरण सहित पूर्ण stepwise समाधान।